ARTICLE NO. 79-4115

On Single-Degree-of-Freedom Flutter Induced by Activated Controls

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It is shown that activation of the trailing-edge control of an airfoil leads to single-degree-of-freedom type instabilities which span over a very wide region of reduced frequencies k, including high values of k (unlike the nonactivated system). These instabilities are shown to be sensitive to changes in pitching axis location, control deflection phase angle, and values of the reduced frequency. These sensitivities of the single-degree-of-freedom system cause the activated airfoil to be potentially sensitive to changes in flight conditions, and may be the source of the many difficulties encountered in suppressing classical multi-degree-of-freedom flutter by means of active controls. The results presented herein relate to zero Mach number and to a 20% trailing-edge control surface.

Nomenclature

b	= semichord length
C	= control gain [see Eqs. (1) and (8)]
h	= displacement of the quarter-chord point, positive downward
I	= torsional moment of inertia per unit span
Im( )	= the imaginary part of ( )
k	= reduced frequency $=\omega b/v$
K	= structural stiffness
L	= aerodynamic lift force, positive downward
M	= aerodynamic pitching moment, positive nose up
L_h,M_h	= oscillatory lift and moment coefficients due to plunging oscillation
L_α,M_α	= oscillatory lift and moment coefficients due to pitching oscillation
L_δ,M_δ	= oscillatory lift and moment coefficients due to control oscillation
R_α	= cross moment of inertia (between torsional and control rotation degree of freedom)
v	= flight velocity
$X_A \cdot b$	= distance of pitching axis from the midchord point, positive downstream
α	= angle of attack
δ	= deflection of the trailing-edge control surface, positive downward
ω	= frequency of oscillation
ψ	= phase angle between α and δ or h and δ [see Eqs. (1) and (8)]
ρ	= air density

Subscripts

R	= real part of the associated parameter
I	= imaginary part of the associated parameter

Introduction

THE classical aeroelastic dynamic instability, known as flutter, is a result of the interaction of two or more structural degrees of freedom. Each of the fluttering degrees of freedom is stable in the absence of the remaining degrees of freedom. Single-degree-of-freedom type flutter instabilities are normally associated with either nonlinear aerodynamics[1] or separated flows.[2] There exists, however, a single-degree-of-freedom type of flutter instability which is based on linear aerodynamics,[3] and comes about when an airfoil oscillates in pitch around an axis located in the vicinity of its leading edge at very low values of reduced frequency k. This instability is known to originate from a negative aerodynamic damping term caused by the unsteady nature of the oscillating flow. This latter pitching instability is, however, of academic nature only, due to the very low values of reduced frequency k required for its existence. In all other cases (having somewhat higher value of k), the aerodynamic damping matrices, due to structural oscillations, are known to be always of positive definite nature.

Recent technological advances in automatic control technology have promoted a considerable number of investigations regarding the effects of active controls on problems of flutter suppression and gust alleviation.[4-6] An active control system on a lifting surface, such as a wing, is designed to actuate a control surface in response to oscillations of the wing in a manner which stabilizes the system. Hence, the activated control surface introduces considerable changes in the aerodynamic forces acting on the system. Since the determination of stability boundaries for multi-degree-of-freedom fluttering system using active controls can be carried out numerically for specific examples only, and since the results obained normally lack in generality, it is proposed to investigate in the present paper the existence of instability boundaries involving activated single-degree-of-freedom systems. Such single-degree-of-freedom instability boundaries can serve to indicate the regions of definite instabilities in any activated multi-degree-of-freedom system, but they clearly fail to indicate the regions of stability for such systems. These simplifed instability boundaries can, therefore, help to define possible regions of stability in complex systems and promote some physical understanding of a complex problem.

Mathematical Model

The airfoil is assumed to oscillate in pitch around an axis located at $x_A b$ from its midchord point (positive direction of displacements and forces are shown in Fig. 1). The trailing-edge control surface deflection is assumed to be driven by a control law of the form:

$$\delta = C\alpha e^{i\psi} \tag{1}$$

Received June 23, 1978; revision received Sept. 15, 1978.
Index category: Aeroelasticity and Hydroelasticity.
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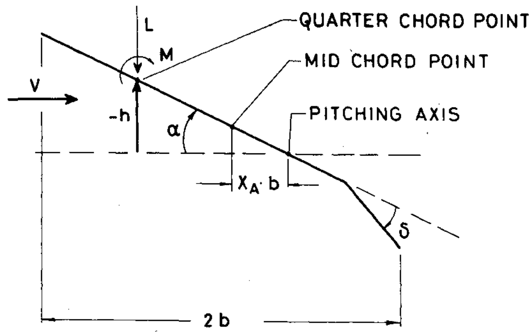

Fig. 1 Description of the two-dimensional oscillating single-degree-of-freedom system.

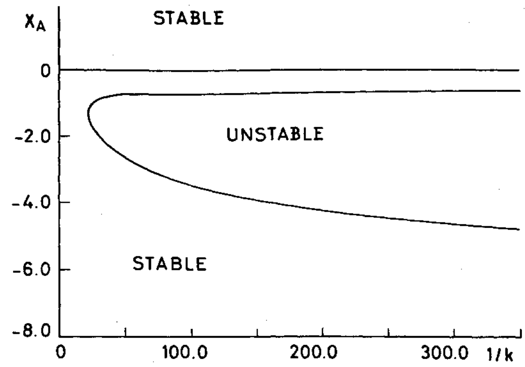

Fig. 2 Instability boundary for the nonactivated single-degree-of-freedom pitching oscillation at $M=0$.

where C denotes the control gain and ψ the phase angle between α and δ. In the absence of structural damping, the equation of motion in the pitching degree of freedom assumes the form

$$I\ddot{\alpha}+K\alpha+R_\alpha\ddot{\delta}=-L(x_A+0.5)b+M \tag{2}$$

where I, R_α, K are inertia, cross-inertia, and stiffness terms, respectively, and L, M are given by[7]

$$L=\pi\rho b^3\omega^2\left[L_h\frac{h}{b}+L_\alpha\alpha+L_\delta\delta\right] \tag{3a}$$

$$M=\pi\rho b^4\omega^2\left[M_h\frac{h}{b}+M_\alpha\alpha+M_\delta\delta\right] \tag{3b}$$

The coordinate h refers to the displacement of the quarter-chord point (positive downward). L_h, M_h, L_α, M_α, L_δ, and M_δ are complex aerodynamic coefficients which depend on k and on the Mach number. For further definition of the notation, see Fig. 1.

Ignoring the inertia coupling with the control surface (R_α), and substituting Eqs. (1) and (3) into Eq. (2) and rearranging, the following equation is obtained using the relation $h/b=-(x+0.50\alpha)$:

$$I\ddot{\alpha}+\{K-\pi\rho b^4\omega^2[(x_A+0.5)^2L_h-(x_A+0.5)$$
$$\times(L_\alpha+L_\delta Ce^{i\psi}+M_h)+M_\alpha+M_\delta Ce^{i\psi}]\}\alpha=0 \tag{4}$$

Remembering that the values of the various aerodynamic derivatives are complex, that I, K are real and positive, and assuming the system to be statically stable, we obtain [from Eq. (4)] the following condition for dynamic instability:

$$\mathrm{Im}[(x_A+0.5)^2L_h-(x_A+0.5)(L_\alpha+L_\delta Ce^{i\psi}+M_h)$$
$$+M_\alpha+M_\delta Ce^{i\psi}]>0 \tag{5}$$

where Im denotes "imaginary part of." It is interesting to note that Eq. (5) contains aerodynamic terms only. For any constant value of Mach number, instability boundaries can therefore be plotted using Eq. (5), for various values of reduced frequency k, of pitching axis locations x_A, of control gains C, and of phase angle ψ.

For the limiting case of pure bending oscillations of an activated control system (with mass balanced control surface), the following equation of motion is obtained:

$$B\ddot{h}+kh=L \tag{6}$$

where

$$L=\pi\rho b^3\omega^2(L_h h/b+L_\delta\delta) \tag{7}$$

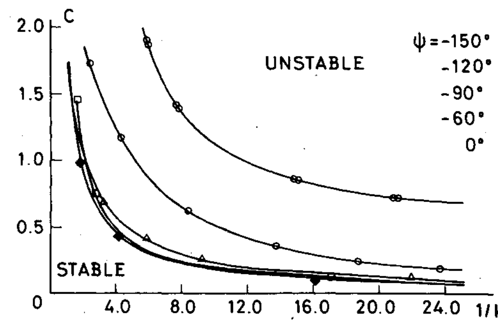

Fig. 3 Instability boundaries for activated system.

Assuming the control law

$$\delta=C(h/b)e^{i\psi} \tag{8}$$

and substituting Eqs. (7) and (8) into Eq. (6), the following condition for dynamic instability in pure bending is obtained:

$$\mathrm{Im}[L_h+L_\delta Ce^{i\psi}]>0 \tag{9}$$

In this case, the instability boundaries (Eq. 9) are functions of k, C, and ψ only (for any given constant value of Mach number).

Presentation and Discussion of Results

The instability boundaries for the single-degree-of-freedom, nonactivated system will first be presented for purposes of subsequent comparison with the activated system. The pure bending instability boundaries of the activated system will then be presented in the form of C vs $1/k$ for various values of ψ. Finally, the pitching instability boundaries of the activated system will be presented in a series of graphs. Each graph relates to a constant value of C and the boundaries are presented in the form x_A vs $1/k$ for various values of ψ. The Mach number is kept equal to zero throughout this work. The system is assumed to have a 20% chord trailing-edge control surface. The aerodyanmic derivativs are computed using analytical expression following the method of Ref. 7.

Instability Boundary for the Unactivated System

Figure 2 shows the unstable region caused by pitching oscillation as a function of the pitching axis location x_A and $1/k$. It can be seen that instability starts around the value of $1/k>25$ or $k<0{,}04$. Furthermore, the critical location of the pitching axis is around the leading edge (that is, $x_A\simeq-1$). The instability boundary in Fig. 2 has been known for many years[3] and it has little practical value due to the very low values of k associated with this instability.

Instability Boundaries for the Activated, Pure Bending Oscillation

Figure 3 shows the instability boundaries C vs $1/k$ for various values of phase angle ψ. The unstable region lies above the various curves, whereas the stable region lies below them. The gain C is made to vary between 0 and 2 and the angle ψ is varied between 0 and ± 180 deg. For negative values of C (i.e., $-2<C<0$), the instability curves shown in Fig. 3 have the form of their reflection (about the abscissa) with the values of ψ changed to $(\psi+180$ deg). This point will be discussed further in a subsequent section of this paper.

It is very interesting to note that:

1) Activated single-degree-of-freedom bending instability occurs over a very wide range of values of k (not necessarily low values of k).

2) Phase angle changes between 0 and -90 deg promote the instability, with $\psi=-90$ deg as the most critical angle. The instability subsides as ψ is further changed toward $\psi=-180$ deg. Positive values of phase angles (0 deg$<\psi<180$ deg) do not show any instabilities within the positive range of values of C, as shown in Fig. 3.

Instability Boundaries for the Activated Pitching Oscillation

Figures 4-11 present the instability boundaries of the activated system. Each figure relates to a different fixed value of gain C and shows the effects of the pitching axis location x_A and the reduced frequency k on the instability boundaries. A careful study of the figures shows that:

1) The instability boundaries cover a very wide range of k values, including a high value of k.

2) The instability regions increase as the gain C is increased.

3) The largest instability regions are obtained for phase angle of $\psi=\pm 90$ deg, with instabilities for both values starting with $C=0.5$.

4) The least unstable location of the pitching axis lies around the midchord region (i.e., $x_A \approx 0$)

5) The phase angles ψ which maintain stability throughout the various values of C and x_A lie in the first quadrant within $0<\psi<30$ deg (that is, in the region of $\psi \simeq 15$ deg).

6) A second range of values of ψ which maintains stability, except for large values of C (that is, $C>1.8$), lies in the third quadrant around $\psi=-180$ deg. For $C=2$ and $\psi=180$ deg, the region of instability is very narow (around the midchord region).

7) For $0<\psi<180$ deg, two distinct regions of instability often occur (see, for example, Figs. 7-11), with one region located at very high values of k (that is, at very low values of $1/k$).

8) The shapes of the instability regions vary considerably with the reduced frequency k. Hence, the employment of unsteady aerodynamics is essential for activated flutter analysis.

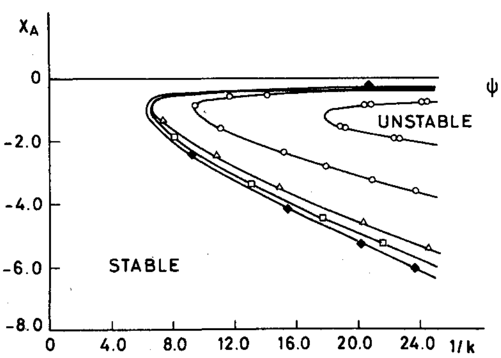

Fig. 4 Instability boundaries for activated system with control gain value of $C=0.30$.

Closed-Form Expressions of the Effects of Control Surface on the Stability Boundaries

It has been shown that in the absence of control surface rotation, single-degree-of-freedom instability can only occur for pitching oscillations provided $1/k>25$ (see Fig. 2). Since the remaining figures presented in this paper (i.e., Figs. 3-11) cover the range of $0<1/k<25$, it follows that instability boundaries within this latter region must be brought about by the detrimental effects of control surface rotation. These detrimental effects can be isolated from Eqs. (5) and (9) to yield closed-form expressions. A study of these expressions

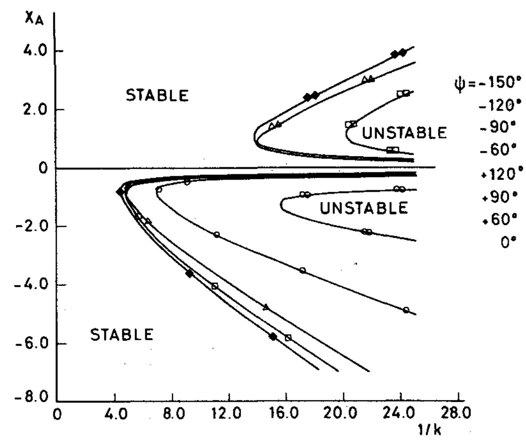

Fig. 5 Instability boundaries for activated system with control gain value of $C=0.50$.

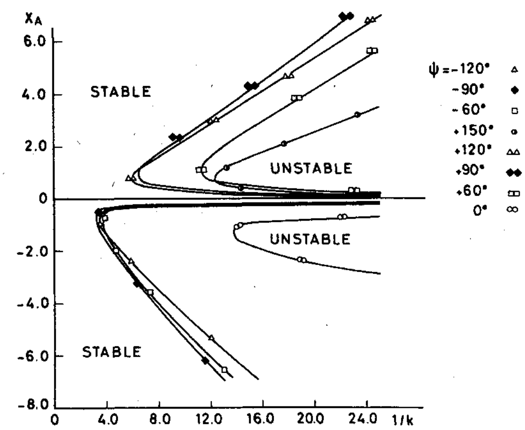

Fig. 6 Instability boundaries for activated system with control gain value of $C=0.75$.

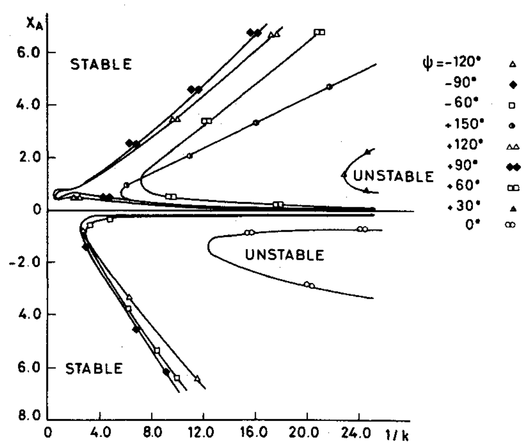

Fig. 7 Instability boundaries for activated system with control gain value of $C=1.00$.

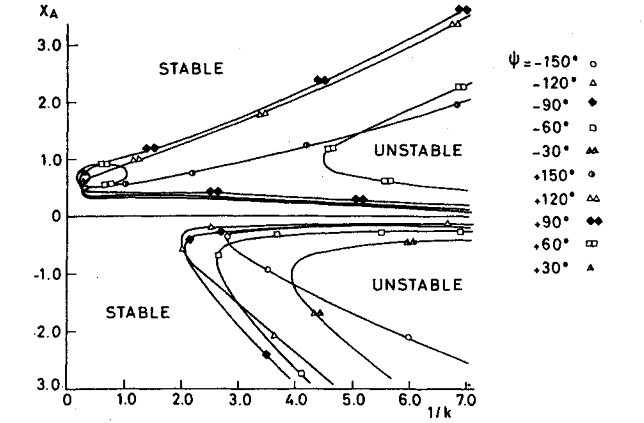

Fig. 8 Instability boundaries for activated system with control gain value of $C=1.25$.

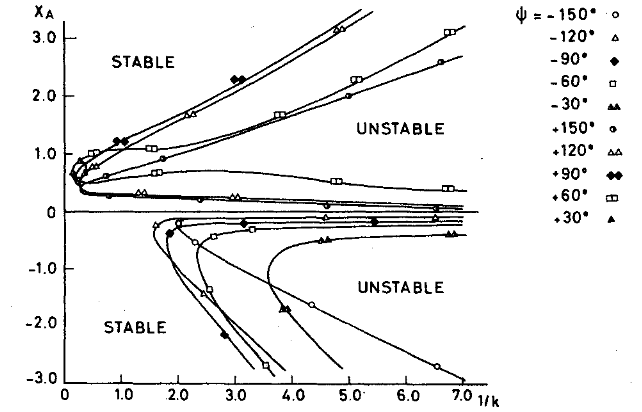

Fig. 9 Instability boundaries for activated system with control gain value of $C=1.50$.

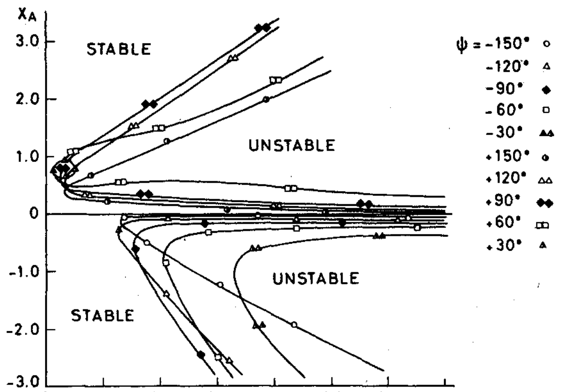

Fig. 10 Instability boundaries for activated system with control gain value of $C=1.75$.

can shed some additional light on the effects of the different parameters and especially on the role of the phase angle ψ.

Control Surface Effects in Pure Bending Oscillations

Equation (9) shows that control surface rotation is destabilizing when

$$\mathrm{Im}(CL_\delta e^{i\psi})>0$$

or, alternatively, when

$$C[L_{\delta_R}\sin\psi+L_{\delta_I}\cos\psi]>0 \qquad (10)$$

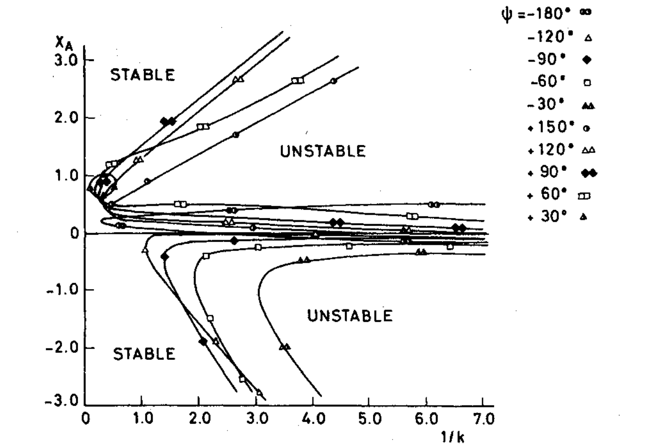

Fig. 11 Instability boundaries for activated system with control gain value of $C=2.00$.

where the added subscripts R and I denote, respectively, the real and imaginary parts of the associated parameters (i.e., L_δ in the preceding case).

The value of L_{δ_R} is about one order of magnitude larger than L_{δ_I} over most of the $1/k$ range (that is, $1/k>1.5$). Hence, instability is largest around $|\psi|\approx 90$ deg for the preceding $1/k$ range. Equation (10) also shows how the real and imaginary parts of the control surface lift coefficient are turned into a pure bending damping coefficient through the phase angle ψ and control gain C. It is worth noting that the following identity:

$$C[L_{\delta_R}\sin\psi+L_{\delta_I}\cos\psi]$$
$$=-C[L_{\delta_R}\sin(\psi+180\,\mathrm{deg})+L_{\delta_I}\cos(\psi+180\,\mathrm{deg})] \qquad (11)$$

implies that instability boundaries with positive gain values may be replaced by identical boundries with negative gain values, provided the corresponding values of the phase angle ψ are increased by 180 deg (as already noted earlier in this work). It may also be observed that the destabilizing effect of the control surface rotation is directly proportional to the control gain C.

Control Surface Effects in Pure Pitching Oscillations

The destabilizing effects of the control surface during pitching oscillations can easily be isolated from Eq. (5) to yield

$$\mathrm{Im}\{Ce^{i\psi}[M_\delta-(x_A+0.5)L_\delta]\}>0$$

or, alternatively,

$$C\{[M_{\delta_R}-(x_A+0.5)L_{\delta_R}]\sin\psi$$
$$+[M_{\delta_I}-(x_A+0.5)L_{\delta_I}]\cos\psi\}>0 \qquad (12)$$

Here again the control surface aerodynamic coefficients are transformed into main surface damping coefficients through the phase angle ψ and control gain C. The inequality expressed by Eq. (12) depends not only on the relative values of L_{δ_R}, L_{δ_I}, M_{δ_R}, M_{δ_I} (which vary with k) but also on the pitching axis location x_A which, in turn, affects the damping of the main surface through the remaining terms in Eq. (5). Hence, the effects of the various parameters on the instability boundaries are of complex nature. Even the dominance of M_{δ_R} over M_{δ_I} is limited to a lower reduced frequency range ($1/k$ greater than about 4) than the corresponding one associated with the L_δ coefficient. Hence, for $1/k>4$, the widest instability will occur when $|\psi| \approx 90$ deg. Figure 11, for example, illustrates this point and also shows that at the lower range of $1/k$ values, the widest instability regions occur with values of $|\psi| \neq 90$ deg.

Some Remarks on Flutter Suppression of Activated Systems

It can be seen that for almost any chosen phase angle, there exists a region of pitching axis locations for which single-degree-of-freedom instability exists. This implies that an activated trailing-edge control may stabilize a mode whose pitching axis lies outside the unstable region and yet may lead to a severe instability of another mode whose pitching axis falls within the unstable region. Similar sensitivities to changes in phase angles can also be observed (keeping the pitching axis location x_A constant), especially in the low region of $1/k$ or high region of k. These facts make stabilization both difficult and also very sensitive to modal and phase angle changes. It is well known that activated flutter suppression systems have a tendency to be sensitive to changes in flight conditions and flight configurations, in addition to their possible adverse effects on initially stable modes. It is, therefore, very possible that this sensitivity essentially originates from the aforementioned single-degree-of-freedom instabilities rather than from the more complex multi-degree-of-freedom flutter.

It is also well known that the classical bending-torsion type of flutter is caused by the skew symmetric components of the real part of the generalized aerodynamic matrix.[4] It can be shown that symmetry in the preceding matrix can be achieved if $C \simeq 1.85$ and $\psi = 180$ deg for $k = 0.1$ or if $C \simeq 2$ and $\psi = 180$ deg for $k = 0.5$. Therefore, classical flutter will not occur for values of C equal to those just specified (dependent on k). Hence, from classical flutter point of view, ψ should lie in the third quadrant, around $\psi = 180$ deg. As already noted, the region of $\psi \simeq 180$ deg leads to single-degree-of-freedom-type instability for values of $C > 1.8$ and is inferior to the first quadrant values from the point of view of the single-degree-of-freedom-type instability. Hence, if C is limited to a value of $C < 1.8$ and $\psi = 180$ deg, no single-degree-of-freedom flutter will occur, but classical flutter may occur. If, on the other hand, C is given the value of 1.85 or larger depending on k, no classical bending-torsion flutter can occur, but a single-degree-of-freedom instability will take place. Hence, a system may exist (having x_A around the midchord region), for which stabilization by means of activated trailing-edge control surface is impossible. The stabilization of such systems can only be achieved if modal changes are introduced that cause the pitching axis to shift from the midchord region. These results are in agreement with those obtained by the use of the aerodynamic energy concept.[8]

Conclusions

It has been shown that activation of the trailing-edge control of an airfoil leads to single-degree-of-freedom-type instabilities which span over a very wide region of reduced frequencies k, including high values of k (unlike the nonactivated system). The origin of these instabilities lies in the introduction by the control surface of negative aerodynamic damping forces. This implies that aerodynamic damping forces must never be neglected while performing flutter analysis of activated control systems (unlike many instances in nonactivated flutter problems). Futhermore, since the instability boundaries vary considerably with the reduced frequency k, oscillatory aerodynamic coefficients must always be used in active control flutter analysis. The sensitivities of the activated single-degree-of-freedom system to changes in pitching axis location, control deflection phase angle, and values of the reduced frequency cause the activated airfoil to be potentially sensitive to changes in flight conditions and may be the source of the many difficulties encountered in suppressing flutter by means of active controls. Since incompressible flow has been assumed throughout this paper, it is felt that further work is required to determine the possible effects of compressibility on the single-degree-of-freedom instability reported herein.

Acknowledgments

This work is part of a study supported by NASA under Grant NSG-7373.

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